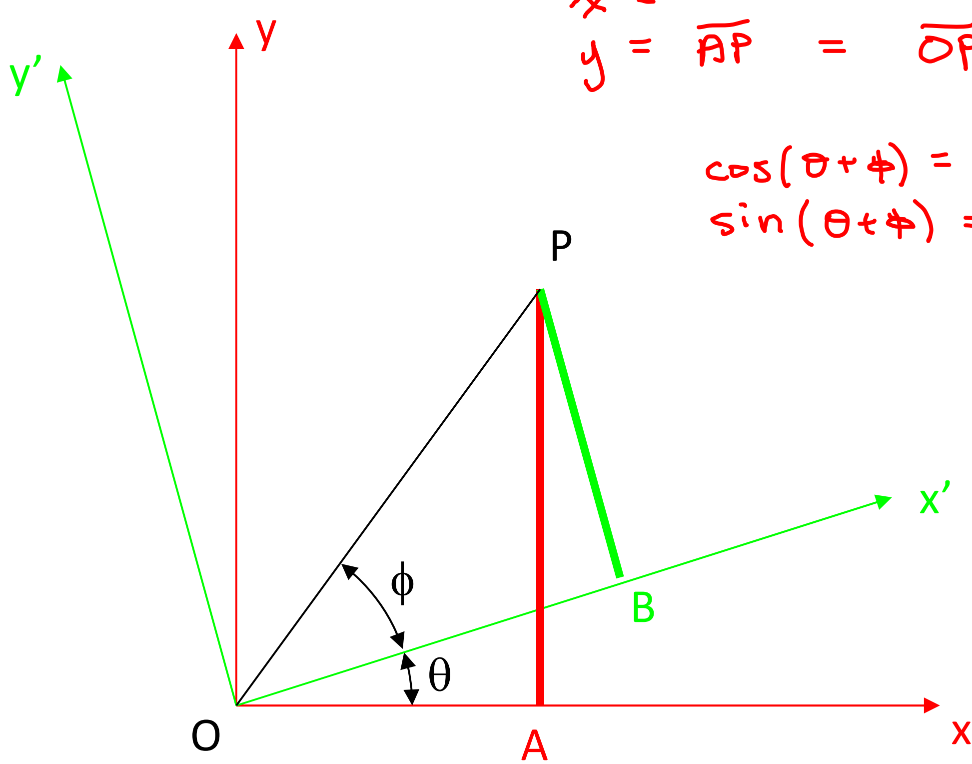
Rotations in 2D



$$x = \overline{OA} = \overline{OP}\cos(\theta+\phi)$$
$$y = \overline{AP} = \overline{OP}\sin(\theta+\phi)$$

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\phi\sin\theta$$
$$\sin(\theta+\phi) = \sin\phi\cos\theta + \cos\phi\sin\theta$$

So

$$x = \underbrace{\overline{OP}\cos\phi}_{x'}\cos\theta - \underbrace{\overline{OP}\sin\phi}_{y'}\sin\theta$$
$$= x'\cos\theta - y'\sin\theta$$

SIMILARLY

$$y = x'\sin\theta + y'\cos\theta$$

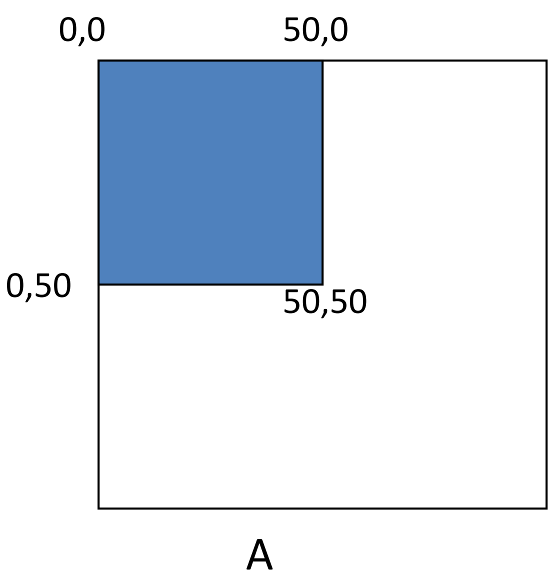
So
$$\begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{R}\begin{pmatrix} x' \\ y' \end{pmatrix}$$

Example

- Image "A" is modified by the affine transform below. Sketch image "B"

$$\begin{pmatrix} x_B \\ y_B \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 1.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_A \\ y_A \\ 1 \end{pmatrix}$$

$x_A\ y_A$		$x_B\ y_B$
(0,0)	$\rightarrow$	(0,0)
(50,0)	$\rightarrow$	(50, 12.5)
(0,50)	$\rightarrow$	(0, 75)
(50,50)	$\rightarrow$	(50, 87.5)



$\rightarrow$